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Interest rates calculation in certain ordinary annuities

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Abstract. Certain annuities are annuities whose payments occur on fixed dates; while a certain ordinary annuity is one in which payments are made at the end of each established period. The calculation of the interest rate, which governs the certain ordinary annuity, involves the use of a non-analytical equation that requires the application of numerical techniques to obtain the value of the aforementioned rate. The literature indicates that any of these techniques requires one or several numerical values for initialization, which generally are estimated using trial techniques, graphical methods or values present in pre-established tables. Through this article, a new robust methodology is proposed that calculates the useful numerical values to initialize the linear interpolation technique, which is used to calculate the interest rate linked to the certain ordinary annuity. The proposed methodology generates initialization values, one by default and the other by excess, which allow us to limit the value of the certain ordinary annuity interest rate considered. Finally, we generated a new strategy that constitutes a novel mathematical model for interest rates calculation in the context of certain ordinary annuity. The percentage relative error obtained indicates the excellent performance of the aforementioned mathematical model.

1. Introduction

The interest rate is a price, the price of credit and its level represents currently, the relative scarcity of capital in an economy. The offer consists of the amount of real money and the savings of the community, that part of the national income that is not spent on consumption. The demand comes from economic agents who seek resources to finance their investments and from those who want to supplement their income to finance consumption levels that they cannot afford with their own resources.

An annuity is, by definition, the set of payments made in equal intervals of time. Certain annuities are annuities whose payments occur on fixed dates; while a certain ordinary annuity (COA) is one in which payments are made at the end of each established period [1].

It is a known fact that if the money is invested or financed, during a period of time and at a certain interest rate, it generates a profit or interest. Now, both business and academically there are several alternatives to calculate the aforementioned interest rate.

The main problem is the ways that the interest rate, that governs the COA, is calculated. It involves the use of a non-analytical equation that requires the application of non-analytical techniques (numerical methods) to obtain the value of the aforementioned rate. In this context, all the numerical methods has as weak feature the starting point (Sp) generation. This Sp is selected without any scientific criterion and it is usually generated using random or arbitrary mechanisms [2].

The more important ways, reported in literature for interest rate calculation linked to COA, are presented at next. In this sense, as a first alternative, people use pre-defined financial tables for the development of problematic situations related to interest rate [3-5]. The main limitation of this method lies in the fact that it allows the calculation of interest rates only for the parameters that appear tabulated, but if the "data" of the problematic situation to be resolved do not correspond to the values contained in them, this alternative it cannot be applied in all possible financial scenarios.

A second alternative is Baily's Formula which establishes the following considerations: In the first instance, the aforementioned formula is only suitable for one of the 16 types of annuities that exist, which are COA. Second, it is valid when the number of investment periods (n) is integer and less than or equal to 50. That is, if the rent is not integer or n is fractional or greater than 50 this alternative is not applicable [6].

The scoring method represents another option for calculating interest rates. This method is based on an "initial guessing" or arbitrary initial guess on the part of the operator who must calculate the interest rate [3-7]. Since the assumption is arbitrary, it lacks justification and financial basis and that is its main limitation.

A fourth alternative is represented by the use of financial calculators. Its main limitation is that it gives wrong answers when n is fractional. This fact can be verified by the resolution of manual exercises with this particularity.

As can be seen, there is no adequate mathematical method or model that efficiently and effectively determines the value of interest rates in COA, which is the fundamental core of the problem posed in the present investigation [3].

Faced with this reality, in this article, it is proposed to use a hybrid model consisting of a novel equation (new mathematical model) for initials values of interest rates in COA and a classical numerical method called linear interpolation (LI) that respond to this uncertainty, through which the calculation of interest rates can be established objectively and at the lowest possible time.

The LI method requires two initialization values (starting points), one by default (I_{def}) and the other by excess (I_{exc}), necessaries for the interest rate calculation.

The main contribution of this paper is the generation of a novel mathematical model for the interest rate calculation linked to COA.

2. Materials and method

2.1. Understanding the certain ordinary annuity mathematical model
A mathematical model that governs the COA is given by Equation (1).

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right],\tag{1}$$

where: P the current value (Capital), A payment per period (Income), n the time interval of the COA and i the interest rate of the COA.

According with the reference [2], this mathematical model represents a non-analytical equation. So, it is impossible to calculate the interest rate using the classical analytic methodologies. For this reason, in this paper, we development a new strategy oriented to interest rate calculation. This strategy is explained at next.

If we separate the terms of the right side of Equation (1), we obtain the Equation (2).

$$P = A\left[\frac{(1+i)^n}{i(1+i)^n} - \frac{1}{i(1+i)^n}\right] \tag{2}$$

The Equation (2) is useless for isolating the interest rate in the first denominator. Now, we can apply the simplification technique for obtaining Equation (3).

$$P = A\left[\frac{1}{i} - \frac{(1+i)^{-n}}{i}\right] \tag{3}$$

Equation (3) exhibits a simpler structure due the time interval of the COA has been cancelled in both denominators. At next, this equation will be modify doing use of arithmetic subtraction for generating the Equation (4).

$$P = \frac{A[1-(1+i)^{-n}]}{i} \tag{4}$$

In this equation, the unique denominator presents only the interest rate which facilities the generation of a function that depend implicitly of this rate. The Equation (4) is transformed applying some basics algebraic transformations for obtaining the Equation (5). In this equation, the right side has not denominators.

$$\frac{i*P}{A} = [1-(1+i)^{-n}] \tag{5}$$

Finally, we construct a real function which independent variable is the interest rate generating the Equation (6). This equation is fundamental for starting point of linear interpolation method.

$$f(i) = \frac{i*P}{A} - [1-(1+i)^{-n}], \tag{6}$$

being: $f(i)$ f definition whose domain is given by the real numbers between 0 and 1.

2.2. Obtaining the mathematical models for interpolation initialization (I_{def} and I_{exc} values)

Here, I_{def} and I_{exc} values match with the starting points. In order to obtain these Sp we use elements of the optimization theory [8-11], generating Equation (7). Being: $f'(i)$ the first derivative of f(i).

$$f'(i) = \frac{P}{A} - [n*(1+i)^{-n-1}] \tag{7}$$

The Equation (7) is vital for generation of mathematical model for calculating the interest rate of COA. Additionally, if Equation (7) is equaled to zero and we make some algebraic manipulations Equation (8) can be obtained. This equation represents the mathematical model for the value by default of the interest rate (I_{def}).

$$I_{def} = \left(\frac{P}{A*n}\right)^{\frac{1}{(n+1)}} - 1 \tag{8}$$

Notice that the I_{def} mathematical model only depend of the know parameters P, A and n. Then, in order to obtain the mathematical model for calculating the value by excess of the interest rate (I_{exc}), it

is possible estimating a novel constant (K) for I_{exc} calculation (see result section). So, we generate the Equation (9).

$$I_{exc} = K * I_{def}$$
(9)

2.3. The mathematical models for linear interpolation method

Linear interpolation is a method that originates from Newton's general interpolation [7]. Linear interpolation method (LIM) allows you to determine by approximation an unknown value that is between two numeric values given. It is also applied to approximate functions, where the values f (a) and f (b) are known and we want to know the one of f(i) included between the two previous values. The LIM is governed by the Equation (10) and it is adequate for the interest rate calculation [8].

$$i = I_{def} + m * I_{exc} - I_{def},$$
(10)

where: i is the interest rate, m is the interpolation line slope, which is given for Equation (11).

$$m = \frac{f(I_{exc}) - f(I_{def})}{I_{exc} - I_{def}}$$
(11)

In Equation (11) m is constant and this property let us to estimate interest rate by linear interpolation.

3. Results

3.1. Obtaining the constant for Iexc calculation

Table 1 shows a partial two-dimensional (2D) view (only 15 observations) about the data used for K calculation. For this, 5000 observations that include P, A, n and id data, were considered for K estimation. Constant K is the arithmetic media of partial constants obtained using these observations.

Table 1. Partial datasheet for K calculation.

P ($)	A ($)	N (months)	id	i_data	Partial Constants (Kj)
75000.00	7500.00	30	0.036075	0.093076	2.580101
350000.00	62000.00	6	0.008747	0.017751	2.029450
10000.00	683.95	36	0.024652	0.060000	2.433877
10580.00	1000.00	14	0.018848	0.040000	2.122216
58300.00	3411.65	24	0.013679	0.029167	2.132305
3306.33	350.00	16	0.031481	0.070000	2.223554
262363.07	5000.00	180	0.006834	0.018333	2.682793
60000.00	2866.66	30	0.011681	0.025000	2.140299
279277.70	10200.00	30	0.002952	0.006000	2.032491
338988.28	10498.00	36	0.002943	0.006000	2.038602
650000.00	12856.31	144	0.007245	0.018333	2.530585
110000.00	5815.82	24	0.009572	0.020000	2.089486
29844.84	2500.00	15	0.014373	0.030000	2.087265
400.00	27.50	18	0.011279	0.023500	2.083599

All the information showed in Table 1 let us to calculate the K value considering the mean value of partial constants Kj. In this sense, the K value is equal to 2.682793 and then the Equation (9) is transformed in the Equation (12) simply changing the K letter by its value. This equation does possible the I_{exc} calculation.

$$I_{exc} = 2.682793 * I_{def} \qquad (12)$$

Additionally, Table 2 shows a partial 2D view (only 15 observations) about the interest rate calculations considering Iexc, Idef and lineal interpolation method. The quantitative information showed in Table 2 is a model based in evidences very important for the confirmation of the interest rate calculated by the strategy proposed, in this paper. Finally, considering the interest rate obtained, by the proposed technique, and the idata, the percentage relative error was calculated generating a value of 1.98%. This result means that the real value of interest rate and the interest rate value calculated, in this paper, are in high correspondence. So, we can say that the strategy proposed exhibits a high performance when the interest rate linked to COA is required. In this way, the Equation (12) is a fundamental base for generating the two starting points necessary for linear interpolation initialization [12].

Table 2. Partial datasheet for interest rate calculation, using LIM.

Idef	Iexc	F (Idef)	F (Iexc)	m	i by LIM	idata
0.036072	0.096780	18.146912	9.686076	-139.373928	0.094500	0.093100
0.008747	0.023465	5.820520	5.536503	-19.296114	0.017800	0.017800
0.024652	0.066136	23.683669	13.612638	-242.767536	0.062000	0.060000
0.018848	0.050565	12.204963	9.862924	-73.840225	0.040900	0.040000
0.013679	0.036696	20.341195	15.775924	-198.332346	0.030100	0.029200
0.031481	0.084457	12.420045	8.604635	-72.021175	0.072800	0.070000
0.006834	0.018333	103.386011	52.473446	-4427.390993	0.018300	0.018300
0.011681	0.031336	25.184877	19.266075	-301.118580	0.025800	0.025000
0.002952	0.007919	28.669492	26.609151	-414.749544	0.006100	0.006000
0.002943	0.007895	34.110861	31.229169	-581.832734	0.006100	0.006000
0.007245	0.019435	89.219032	48.23377	-3361.896973	0.018700	0.018300
0.009572	0.025678	21.352086	17.751561	-223.534556	0.020500	0.020000
0.014373	0.038559	13.407117	11.231224	-89.962839	0.030700	0.030000
0.011279	0.030258	16.208202	13.723555	-130.912236	0.024000	0.023500

4. Conclusions

Through this article, a new robust strategy is proposed that calculates the starting numerical values to initialize the linear interpolation technique which is used to calculate the interest rate linked to the COA. For this, optimization theory elements and linear interpolation technique were used in order to obtain the K value necessary for estimating the default and excess values of interest rate.

The main advantage of this strategy is the generation of the mathematical model, for calculation of Sp, which is given by the Equation (12). This equation is not reported in the specialized literature and, for this reason, it constitutes a novel model for the Sp calculation not only for linear interpolation but for any other numerical method that requires two starting points and that can be used in the financial mathematics context.

The percentage relative error obtained indicates the excellent performance of the proposed strategy due the idata (see Table 2) and the interest rate calculated yielded comparable values to each other.

In the future, it is planned to extend the proposed strategy for interest rate calculation linked to another types of annuities in order to estimate the robustness of the aforementioned strategy.

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