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Developmental perspectives of numerical thinking for the interpretation of physical quantities

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Abstract. This research reveals the perspectives in the teaching of numerical thinking through a documentary review. The representation of physical elements such as functions, vectors and operators, and their subsequent interpretation through numerical thinking gives meaning to the physical quantities that such elements represent. A documentary sample integrated for 40 sources on numerical thinking such as articles published in indexed journals, postgraduate dissertations, and books is considered. A qualitative content analysis method is used. First, an encoding procedure is applied for tagging the extracted information from the source documents. Then, a split and merge procedure is considered in order to establish from the tags the dimensions and categories that allow determining the conceptual relationships that support the developmental perspectives of numerical thinking. The method reveals that the numerical thinking can be developed in the global context from four perspectives, namely, historical, theoretical, curricular, and social perspectives. From these results, an incorporation of the such perspectives can be institutionalized for promoting curricular, didactic and evaluative new proposals for numerical thinking teaching.

1. Introduction

The complexity of teaching mathematics in the university context is well known [1], which has led, in recent years, to develop research oriented towards the study of the mathematics learning and teaching approaches and its applications in higher education [2]. Among the aspects that increase the complexity of mathematics teaching is that students at all levels reject mathematics, fear and dislike it, and if given the opportunity, such courses are taken as a last resource. Another aspect, despite not being negative, which has increased the complexity in the teaching of mathematics is the great demand for mathematical knowledge of the knowledge society [3]. In any case, both aspects require academic actors committed to effective mathematical teaching who are responsible for the complex coordination of the teaching processes of specific knowledges with their specific beliefs and culture in the teaching and learning of mathematics in the primary, secondary and tertiary level [4].



The generation, discussion, refutation and transmission of mathematical knowledge in academic institutions traditionally turn around curricular structures designed to articulate the learning context, basic knowledge and intrinsic mathematical processes [5]. Basic knowledge also known as mathematical thinking must be developed by academic actors (teachers/students), such a development process conceived from the systematic perspective presents an academic actor as input and as outputs: the habits of mind development [6], the underlying processes of understanding [7], and the organizational procedures for teaching/learning [8]. Table 1 describes the sub-processes associated with the process of developing mathematical thinking and shows the possible outputs as direct implications of mathematical knowledge.

Table 1. Elements of the process of developing mathematical thinking.

Input	Process	Output
	Developing habits of mind	Defining
		Systematizing
Academic actor	Developing underlying processes of understanding	Abstracting
		Making connections
		Make predictions
		Creating
		Inventing
		Conjecturing
		Imagining
		Expressing
		Specializing
		Conjecturing
Organizational procedures for teaching/learning		Generalizing
		Convincing
		Focusing
		Defocusing
		Sense-making
		Looking back
		Getting unstuck
		Articulation

Mathematical thinking is concerned to a manner of thinking focused on reasoning based on particular operations, processes, and dynamics of mathematical nature [9]. At this point it is important to note that mathematical thinking is supported and acquires perfection from logical thinking [10], the latter defined as the rational cognitive process of actively reflecting objective reality with the help of concepts, judgments, reasoning and other forms of thinking [11]. Mathematical thinking according to the basic standards of mathematics competencies proposed by the Colombian State [12], is subdivided into five types, namely, numerical thinking, spatial thinking, metric thinking, random thinking, and variational thinking. Regarding the numerical thinking, which corresponds to the study object of this paper, it is associated with the processes of understanding the use and meanings of numbers and numbering; understanding the meaning and significance of operations and the relationships between numbers; and the development of different calculation and estimation techniques [13].

Effective mathematical teaching involves approaching the curricular phenomenon and the mathematics teaching/learning process from the perspective of the didactics of science. In this sense, the elements of mathematical thinking must be developed using pedagogical activities of instruction or knowledge with the objective of increasing the standards of effective mathematical teaching; therefore, it is interesting to reveal the current state of teaching/learning processes related to numerical thinking. In this work, a documentary research is considered in order to reveal the developmental perspectives of numerical thinking; the documentary research is supported by a qualitative content analysis method used to establish the relationships between elements of the process of developing mathematical thinking, such relationships are then considered for elaborating certain reflections and considerations about the numerical thinking teaching/learning processes. These reflections and considerations to give support to developmental perspectives of this mathematical thinking.

2. Materials and methods

Since the objective of the research is to propose a perspective of the development of numerical thinking based on existing teaching/learning standards and approaches, the research process requires a qualitative approach [14], since it is necessary to study the characteristics and qualities of the didactics of numerical thinking. The content analysis study as a qualitative research method focused on performing a detailed and systematic exploration of the contents of a set of documentary sources in order to identify patterns, themes, or bias [15], is considered.

A theoretical sample of 40 documentary sources composed of journal papers, postgraduate dissertations and books, is considered for the content analysis study. The information extracted from the documents is systematized in an analysis matrix and then coded using the procedure proposed in [16]. The idea is used these tagged information as the input to a split and merged procedure which output corresponds to the structural elements of a semantic network. The merge procedure is incorporated because in some cases the descriptive coding scheme requires to merge synonymous codes into one resulting representing them, which replaces the merged codes and receives all of their links to other codes. While, the split procedure is considered in order to split a code into multiple sub codes. This integration into a complex structure network the information tagged in the analysis matrix is performed using “Atlas.ti” [17]. From the final semantic network, a set of reflections and considerations are generated that support the developmental perspectives of numerical thinking.

3. Results

The content analysis allowed to generate the set of codes used to synthesize a semantic network of the developmental perspectives of numerical thinking. This semantic network is presented in two parts. First, the basis for the development of numerical thinking is presented according to the semantic network shown in Figure 1. Then, the semantic structures that reveal the developmental perspectives are shown in Figure 2.

3.1. Bases for the development of numerical thinking

Both mathematical knowledge and types of thinking, as well as the general processes around didactic and the learning context of mathematics, constitute elements in mathematics teaching/learning process. Regarding general processes, the competencies of communicating, reasoning, modeling processes and phenomena of reality as well as formulating, comparing and exercising algorithms must be generated from the mathematics teaching and learning strategies [18]. The contexts, socio-cultural, school and local (classroom) make up the global context in which the academic actor in charge of teaching mathematics performs, hence this actor must take such global context into account [19]. And finally, as it was previously presented in Table 1, the development process of mathematical thinking has direct implications

for mathematical knowledge. Figure 1 shows the semantic network associated to fundamentals of perspectives of the development of numerical thinking.

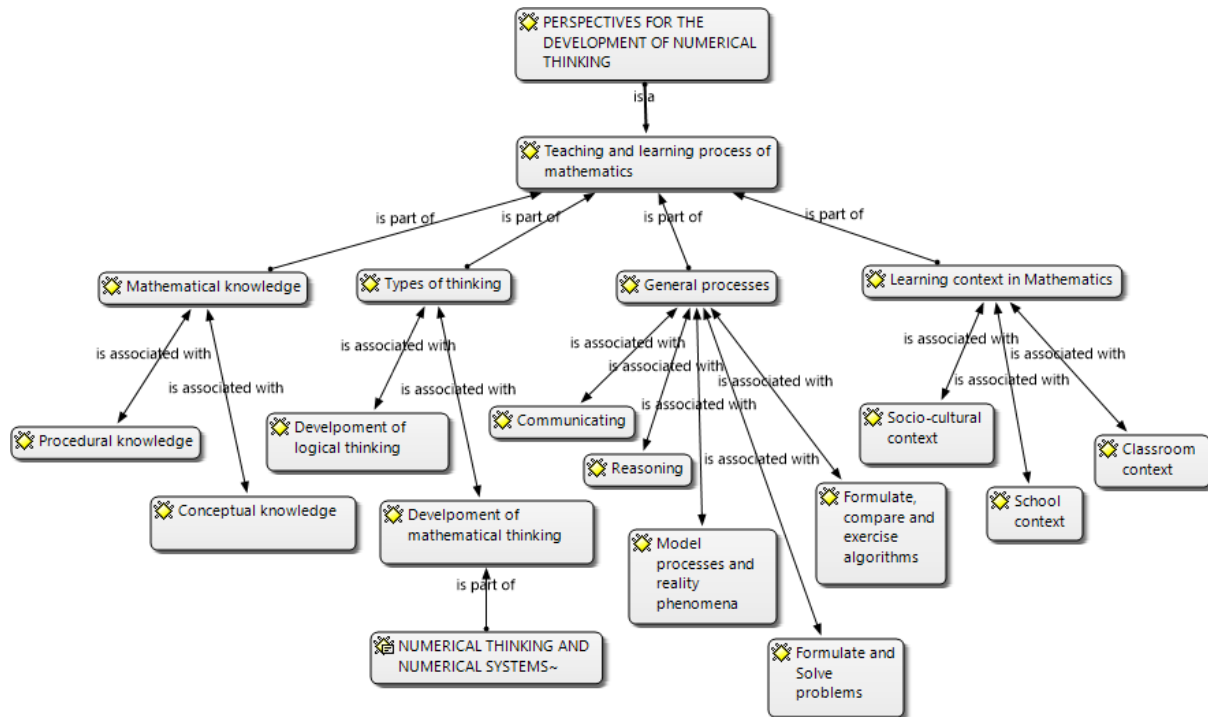


Figure 1. Perspectives for the development of numerical thinking.

3.2. Numerical thinking

As for the development of numerical thinking, the central point of this work, the content analysis derives in a semantic network whose axial categories correspond to: (1) historical and epistemological perspective; (2) theoretical perspective; (3) curricular perspective; and (4) social perspective of knowledge. Figure 2 shows the elements for the development of numerical thinking.

3.2.1. Historical and epistemological perspective The traditionally characteristic of the mathematics known as “mathematical precision” [20], which academic actors in other disciplines have given to mathematics, gives it the connotation of a very special science [21]. This major status is directly linked with its very peculiar epistemology [22], which is associated with mathematical proof as the specific technique of the discipline [23]. In order to promote numerical thinking, it is required to interpret the epistemological criteria considered in mathematical knowledge [24]. Nevertheless, such epistemological principles must be supported by their historical foundations [25].

From the historical perspective, numbering systems [26], such as the Greeks, Romans, Indo-Arabic, another as numbering systems for natural, real, rational and complex numbers, as well as new systems such as the base 4 numbering system, must be mandatorily considered in the pedagogical construct used for establishing the strategies for teaching/learning of numerical thinking [27].

3.2.2. Theoretical perspective The development of mathematical competence associated with numerical thinking by means incorporating of ability to understand, judge, do, and use

theoretical mathematics foundations of this kind of thinking across a variety of mathematical situations, will allow to reach its overarching goal in the mathematical teaching [28].

In the semantic network of Figure 2, the natural interconnection between the historical-epistemological perspective and the theoretical perspective is evident. The theoretical perspective has to do with the theoretical corpus, which is manifested by the approach presented towards logic semiotic [29], the cardinality of quantities [30], the study of magnitudes, quantities and measurements [31], the meaning of numbers and numbering [32], the meaning of operations [33], the notions about incommensurability [34], irrationality [35], completeness [36] and continuity [37], and the relationships between numbers [38].

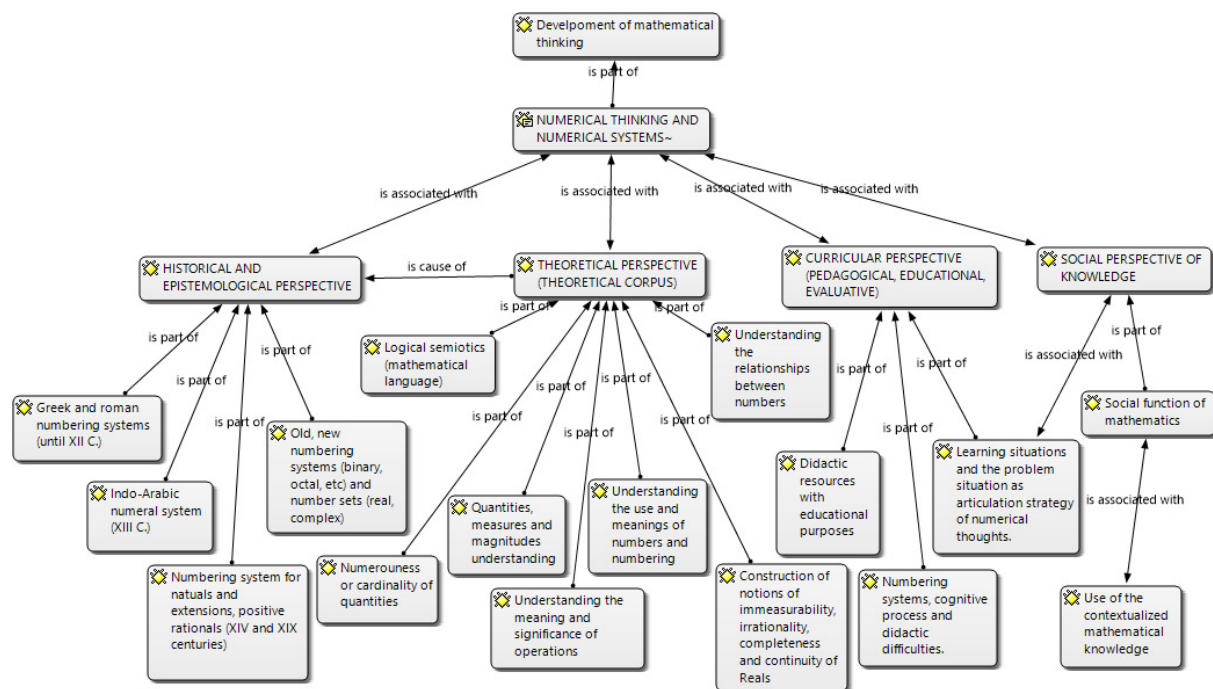


Figure 2. Development of numerical thinking.

3.2.3. Curricular perspective The pedagogy oriented to the understanding and development of the teaching-learning process in educational institutions, considering both the institutional organizational structure and the requirements and characteristics of the educational community, uses didactics to study the processes and elements existing in the educational process [39].

The curriculum is an element of the educational model which is based on a pedagogical theory. This element guides the educational process through the definition of whom to teach, why to teach, what to teach, how, what, when and why to evaluate [40,41]. Additionally, the curriculum allows to understand the context [42], determine the purposes of education, the sequences, the methodological strategies and the evaluation processes in an educational institution [43]. The interaction between teachers and teaching resources for educational purposes shows the effectiveness of the set of elements from the curricular perspective for the development of numerical thinking. Such interaction marks a crossing point between the curricular perspective and the rest of the revealed perspectives of the content analysis [44].

3.2.4. Social perspective of knowledge Training in the set of processes, concepts, propositions, models and theories in various contexts, which configure mathematical conceptual structures and the use of different number systems, allows the development of numerical thinking [45]. Among

the objectives is the articulation of numerical thinking with the learning situations and the problem situation, which associates the perspective of the knowledge social with the curricular perspective [46].

From this perspective, academic actors become social actors. The teacher articulates the educational process between the students, the educational institution and the educational system. In any case, the teacher becomes an axis of the social function of mathematics education [47]. As a social actor, the teacher teaches mathematics as useful, affectionate, profitable, convenient and important knowledge, without deviating from its abstract nature, orienting knowledge towards the solution of problems, giving meaning and meaning of knowledge in its teaching of everyday life. For the teaching of mathematics, the contextualization and location of the student in their context are important, and perhaps more important is the student's ability to socialize the mathematical knowledge acquired before his community, therefore the student also acquires the role of social actor. [48].

4. Conclusions

In this work, a documentary research based on the content analysis method is considered in order to reveal the developmental perspectives for numerical thinking in the process of mathematical teaching. The work presented allows to deepen the knowledge about how numerical thinking should be considered in the teaching/learning process in order to increase the ability to interpret physical quantities. After the content analysis method application, a set of codes is obtained, then this tagged information is analyzed considering a procedure of split and merge of the codes. The results of this stage correspond with the tags and the interconnection branches of the semantic network that defines the developmental perspectives of the numerical thinking. From the semantic network it can be observed that the development of the numerical thinking is associated with four perspectives in the teaching of mathematical thinking. These perspectives are the axial categories of the semantic network, namely historical-epistemological perspective of numerical thinking, theoretical perspective of numerical thinking, curricular perspective of numerical thinking and, social perspective of knowledge of numerical thinking.

The teaching-learning process of mathematics requires of a historical and epistemological perspective for its development. The epistemological principles based on the proof method and the old and new numbering systems support the mathematics thinking teaching. Regarding the theoretical perspective, it is very important the development of disciplinary competencies around the theoretical corpus of mathematical knowledge associated with numerical thinking, namely logic semiotic and the meaning of numbers and numbering, mainly. An important perspective is the curricular. The curricular, didactic and evaluate proposals for the teaching of numerical thinking allow improved the teaching practice, the contextualization of mathematical knowledge, and the incorporation the research processes in the classroom. Finally, the social perspective of knowledge establishes the importance that the numerical thinking have in the mathematical teaching as social function when addressing problems, facts and/or contextualized phenomena.

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