An optimization approach for inventory costs in probabilistic inventory models: A case study

Un enfoque de optimización para costos de inventario en modelos de inventario probabilísticos: Un caso de estudio

Alexander Pulido-Rojano[1]* Andrea Pizarro-Rada[1] Miguel Padilla-Polanco[1]
Milton Sánchez-Jiménez[1] Ladianys De-la-Rosa[1]

Recibido 4 de abril de 2018, aceptado 17 de junio de 2019
Received: April 4, 2018 Accepted: June 17, 2019

ABSTRACT

Inventories represent stocks of goods necessary for operations of sales or manufacturing in a company. These allow to the companies meet their sales levels, while representing an opportunity to the cost control and the decision-making. This paper presents an optimization approach to minimize the inventory costs in probabilistic inventory models of independent demand. The approach has been validated for set the policy optimal of inventories with probabilistic demand within a company that markets disposable products. The established policy aims to minimize the inventory costs by using the standard deviation of the historical data, the mean deviation of forecast errors and the mean deviation of the historical data. For the determination of the economic order quantities, three types of products were selected, taking historical sales data. Likewise, different forecasting methods were used, selecting the one that minimizes the mean squared error for the forecasted demand. The proposed methodology is practical and easy to use in companies where inventories have probabilistic and independent demand. Also, the proposed approach allowed optimize the costs related to holding costs, ordering costs and safety stock costs.

Keywords: Probabilistic inventory models, independent demand, safety stock, forecasting methods, total cost of inventory, dispersion of demand.

RESUMEN

Los inventarios representan la existencia de mercancías necesarias para las operaciones de ventas o fabricación en una empresa. Estos permiten a las empresas cumplir con sus niveles de ventas, al tiempo que representan una oportunidad para el control de costos y la toma de decisiones. Este documento presenta un enfoque de optimización para minimizar los costos de inventario en modelos de inventario probabilísticos de demanda independiente. El enfoque ha sido validado para establecer la política óptima de inventarios con demanda probabilística dentro de una empresa que comercializa productos desechables. La política establecida tiene como objetivo minimizar los costos de inventario utilizando la desviación estándar de los datos históricos, la desviación media de los errores de pronóstico y la desviación media de los datos históricos. Para la determinación de la cantidad económico de pedido, se seleccionaron tres tipos de productos tomando datos históricos de ventas. Asimismo, se utilizaron diferentes métodos de pronóstico, seleccionando el que minimiza el error cuadrático medio para la demanda pronosticada.

[1] Universidad Simón Bolívar. Industrial Engineering Department. Barranquilla, Colombia.
E-mail: apulido3@unisimonbolivar.edu.co; apizarro4@unisimon.edu.co; mpadilla39@unisimon.edu.co; msanchez38@unisimon.edu.co; ldelarosa10@unisimon.edu.co
* Autor de correspondencia: apulido3@unisimonbolivar.edu.co

La metodología propuesta es práctica y de fácil uso en empresas donde los inventarios tienen una demanda probabilística e independiente. Además, el enfoque propuesto permitió optimizar los costos relacionados con los costos de mantenimiento, los costos de pedido y los costos de inventario de seguridad.

Palabras clave: Modelos de inventario probabilísticos, demanda independiente, stock de seguridad, métodos de pronóstico, costo total del inventario, dispersión de la demanda.

INTRODUCTION

Organizations dedicated to the marketing of products usually have some questions concerning the ordering of products necessary. These questions, are usually related to two specific situations. First, in case of shortage (i.e. insufficient products to supply the demand) that causes potential losses due to miss sales opportunities. Secondly, in case of excess inventories that greatly increases the holding and maintenance costs [1-3]. The goal is to balance these two situations having the right quantities of each product and at the same time, avoid the decrease of sales due to lack of goods [4, 5]. To establish the optimal quantities of inventory of a product, the demand must be considered over a period of time (days, weeks, months, and years), however, when we obtain this information, the product can have a constant or variable demand; which would hinder the process of setting the optimal quantities need to order. In any case, the responsible of these decisions aims to create and implement techniques, methods and methodologies that allow good inventory management to accurately determine the quantities to be ordered at the right time [5-7].

In this paper, an optimization approach for minimizing inventory costs in probabilistic inventory models of independent demand is proposed according to the behavior of demand from each product and the inventory model that best fits the company. The standard deviation of the historical data, the mean deviation of forecast errors and the mean deviation of the historical data were taken into account to calculate the inventory total cost. Furthermore, several forecast techniques were also used to establish the future demand values. The study was carried out in a company specialized in the commercialization of disposable product located in the city of Sincelejo-Colombia. This approach has been validated on three different products: Styrofoam dishes, rolls of plastic sheeting and die-cut bags.

This paper has been structured as follows: Section 2 shows a summary of some studies related to inventories topic. Section 3 presents the methodology used in this document to the implementation of the proposed optimization approach. In Section 4, the evaluated and compared forecasting methods to minimize the mean squared error are displayed. The Section 5, presents the measures of dispersion used to determine the security stock of the evaluated inventory policies. Section 6 shows the approach to determining the economic order quantities and the total cost of inventory. The results and analysis of the numerical experiments are presented in Section 7, followed by the conclusions in Section 8.

RELATED WORKS

Inventory models have been recently studied by different authors, who have generated and presented significant approaches. For example, In [8] it was proposed an optimal combination of reserves instances on demand, so that the demand is satisfied and the costs are reduced to the minimum. To achieve this goal, this study presented a stochastic model based on inventory theory and it was formulated as an inventory management problem. In [9] it was proposed a model for the pricing and the inventory control of non-instantaneous deteriorating items. They determined the optimal selling price and the optimal inventory control variables by study of the behavior of the deterioration of the items. In [10] it was developed an inventory control model when replenishment intervals are probabilistic and partial backordering. They considered that inventory at the beginning of the period is not equal to zero and it can be positive or negative amounts, therefore, the goal was to determine the optimum amount of replenish-up-to level in special sale offer. In [11] it was presented an integrated inventory model to find optimal solutions for lot size, setup cost, and the total number of deliveries from the supplier to the buyer in a single production run, so that the expected total cost is minimum. In [12] it was proposed reducing investment in safety stock by using the standard deviation of forecast errors instead of standard deviation of the historical demand. In [13] it was developed an economic order quantity (EOQ) model to determine the joint ordering policy for two products under completion and substitution conditions. In [14] it was developed a deterministic inventory model with ramp-type demand depending on price and time. The aim was to maximize the total profit per unit time. In their approach, they assumed that the cumulative holding cost is a nonlinear function of time and presented a procedure to determine the economic lot size, the optimal inventory cycle and the maximum profit. In [15] it was considered an inventory model under the classical EOQ framework. The author assumed that shortages of inventory affect the customers' demand and, to obtain the total cost, it was approximated the backlogged demand rate by using a piecewise constant function. Finally, in [16] it was presented a unified EOQ model with financial constraints and market tolerance. It considered the EOQ paradigm with partial backorders to study factors related to the financial crisis. In this sense, an exact unified model was developed to calculate the total average profit/cost, the optimal profit/cost and the decision variables.

METHODOLOGY

In this research were determined and compared the total costs in a probabilistic inventory model using three different measures of dispersion of the demand: the standard deviation of the historical data, the mean deviation of forecast errors and the mean deviation of the historical data. Prior to that, three forecasting techniques (moving average method, weighted moving average method and exponential smoothing model) were compared to identify the forecasting method with the lowest value for mean squared error. This ensured that the mean deviation of forecast errors used to calculate the total cost of inventory was the minimum. In the determination of the security stock levels was taken into account the standard deviation of the data for the three specific cases mentioned above. Moreover, the quantity of goods to request was fixed based on three components: the costs of holding, ordering and security stock.

As a study case to demonstrate the usefulness of the proposed approach, three kind of products were chosen from a company specialized in the commercialization of disposable product: Styrofoam dishes Nº 20, rolls of plastic sheeting (6 meters wide and 450 meters long per roll) and die-cut bags (0.305 meters wide and 0. 4064 meters long per bag) (see Figure 1). The historical monthly demand of the last four years, as well as the order cost, lead time and holding cost per product are shown in the Tables 1 and 2.

FORECASTING METHODS

Forecasting methods are useful aid in decision making under scenarios of uncertainty [17, 18]. They reduce uncertainty about the future, allowing the implementation of actions in line with the organization's best interests [18-20]. As it has been mentioned, three forecasting techniques were evaluated and compared (moving average method, weighted moving average method and exponential smoothing model) to identify the technique that minimizes the

Figure 1. Styrofoam dishes, rolls of plastic sheeting and die-cut bags.

Table 1. Unit costs and lead time per product type.

	Styrofoam dishes (Package of 20 units)	Rolls of plastic sheeting (roll)	Die-cut bags (Package of 100 units)
Lead Time (days)	8	8	8
Ordering Cost (per order)	$1.200	$4.000	$10.000
Holding cost (per unit per day)	$1.94	$100.33	$3.34

Table 2. Historical monthly demand for Styrofoam dishes, rolls of plastic sheeting and die-cut bags.

Month	Year	Styrofoam dishes (Package of 20 units)	Rolls of plastic sheeting (roll of 6 meters wide and 450 meters long)	Die-cut bags (Package of 100 units)	Year	Styrofoam dishes (Package of 20 units)	Rolls of plastic sheeting (roll of 6 meters wide and 450 meters long)	Die-cut bags (Package of 100 units)
January	2014	3600	18	280	2016	3000	12	200
February		2784	12	200		2520	10	150
March		2400	15	130		2300	14	100
April		3000	17	140		3000	15	120
May		3000	20	180		2808	18	150
June		2880	15	150		2515	11	120
July		2400	15	120		2400	10	100
August		3600	16	120		3600	12	120
September		3144	19	220		3120	14	170
October		3120	18	160		3214	10	150
November		4080	20	190		3815	19	180
December		3840	24	260		4500	20	190
January	2015	3120	15	180	2017	3360	14	240
February		2760	11	170		2760	11	160
March		2060	12	90		2110	18	115
April		2808	15	130		2760	12	150
May		2800	14	150		3000	15	180
June		2664	14	140		2635	15	110
July		2304	13	110		2520	16	160
August		3480	11	90		3360	11	100
September		3100	15	200		3240	20	180
October		3150	12	130		3070	15	140
November		3835	16	185		3840	18	200
December		3646	22	200		4675	25	250

mean squared error and so, use it to calculate the total cost of inventory and the security stock. This allow us compare the results with the values obtained by the standard deviation of the historical data and the mean deviation of the historical data (see later section 7). A summary of the forecast methods used in this research are presented below.

- ***Moving Average Method (MA):*** It is a technique to get an overall idea of the trends in a set of n data from the last n periods. It can be calculated for any period of future time and any subset of data [18]. The MA uses the average of the k + 1 most recent data (in our case, the k + 1 most recent historical data of demand) in the time as a forecast for the next period and it can be calculated as follows:

$$F_{n+1} : \frac{x_n + x_{n-1} + \ldots + x_{n-k}}{k+1} \tag{1}$$

Where x_i is the actual value of the demand for the period i, $\{i = 1 \ldots n\}$. "Moving" indicates that, while a new observation is available, it replaces the oldest observation, and a new average is calculated. As a result, the average will change, as new observations emerge [18-20].

Weighted Moving Average Method (WMA): It is a forecasting technique which assign a heavier weighting to the most recent data than the oldest data. It is based on the principle that recent data are more relevant. Each observation x_i in the calculation receives a different weight β_i, and the sum of the weights β_i must be equal to 1 [17, 20]. WMA also uses the $k + 1$ most recent historical data of demand to calculate the forecast for the next period, as follows:

$$F_{n+1} = x_n \cdot \beta_n + x_{n-1} \cdot \beta_{n-1} + \ldots + x_{n-k} \cdot \beta_{n-k} \qquad (2)$$

Where $\beta_n \geq \beta_{n-1} \geq \ldots \geq \beta_{n-k}$ *and* $\sum_{i \in k+1} \beta_i = 1$.

- ***Exponential smoothing model (ES):*** It is a technique appropriate for forecasting data with no trend and is also suitable when there is little data available. In this model, the forecast for the next period F_{n+1} is equal to a weighted average between the most recent observation x_n and the most recent forecast F_n [19, 21], so that it can be calculated as follow:

$$F_{n+1} = \theta x_n + (1-\theta) F_n \qquad (3)$$

Where $0 \leq \theta \leq 1$ is the smoothing parameter.

These methods have been proposed according to the characteristics of the demand of the studied products (although this will depend, in other contexts, on the type of demand). The accuracy of each technique is tested by calculation of the mean squared error (MSE); which measures the average of the squares of the errors between the forecasted demand and its corresponding actual demand for t forecasted periods:

$$MSE = \frac{\Sigma_{i \in t} (x_i - F_i)^2}{t} \qquad (4)$$

DETERMINATION OF THE SECURITY STOCK

It is well known that there are different types of inventory models to establish the optimal quantities, and the use of them depends on the type of demand of the product concerned [2, 22-25]. According to authors such as [2], the demand can have a probabilistic behavior when the coefficient of variation of the historical demand (*CV*) is equal or greater than 20%, that is:

$$CV = \frac{\sigma_s}{\bar{x}} x100 \qquad (5)$$

where; $\bar{x}$ is the average of the historical demand and σ_s is its standard deviation. Probabilistic inventory models take into account that the critical period in one inventory cycle is related to the waiting period or lead time (*L*), because during this time there might be an inventory shortage. To avoid these eventualities, the ideal is to have an additional amount of goods as a security stock (*ss*). So, if the historical demand follow a standard normal probability distribution $N(\mu_L, \sigma_L)$ during the lead time, then the security stock will depend on the critical value (Z_a) of the standard normal probability distribution $N(0,1)$ for a significance of a. The security stock (*ss*) is set in such a way that the probability of shortages during L is at most a, this is:

$$P\{x_L \geq ss + \mu_L\} \leq \alpha \qquad (6)$$

where x_L is the actual demand during the period L. This is equivalent to $P\{z \geq ss/\sigma L\} \leq \alpha$, where $z\ (x_L - \mu_I)/\sigma_L$. In this way, to obtain the minimum stock quantities it must be met $P\{z \geq z_a\} \leq \alpha$, and therefore, $ss \geq \sigma_L \cdot z_a$ [2, 12].

An important part of the proposed approach is to determine the levels of security stock which get a minimum total cost and, at the same time, guarantee the levels of services desired by the company. In this sense, it's proposed to establish an inventory policy where the security stock will depend on three different ways of calculating the dispersion of the demand; which is subsequently adjusted to L periods, so that we can finally obtain the S_L value.

- The standard deviation of the historical data:

$$\sigma_s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} \qquad (7)$$

- The mean deviation of forecast errors (Best forecasting method):

$$\sigma_f = \sqrt{\frac{\pi}{2}} \cdot \frac{\Sigma_{i \in t} |x_i - F_i|}{t} \quad (8)$$

- The mean deviation of the historical data:

$$\sigma_\alpha = \sqrt{\frac{\pi}{2}} \cdot \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n} \quad (9)$$

In this way, the values of the dispersion are used to determine the security stock with a desired service levels. Authors such as [12] have shown that the mean deviation of forecast errors is a recommended method to products with probabilistic demand, however, this may not always be the case. In our approach, it's compared the total cost of inventory in each case, identifying the best policy for the company (see section 7).

ORDER QUANTITY AND TOTAL COST

The company targeted in this case study set its optimal quantities by calculating the average historical sales. At present, the organization makes a single monthly order for the quantities that it expects to sell in this period. Obviously, this policy does not take into account losses due to shortages or holding costs, being inefficient in terms of profitability. It's proposed to evaluate two inventory policies which take as input the demand dispersion in each case displayed section 5. The first, where a monthly order is made for each product and, the second, an annual policy where the quantities to be ordered are:

$$Q = \sqrt{\frac{2DC_o}{C_h}} \quad (10)$$

Here, D is the expected annual demand, C_o is the ordering cost per order and C_h is the holding cost per unit per year. However, the approach to calculating the economic order-quantity must be adjusted to the context of the organization.

The annual policy uses an economic order-quantity model and assumes that the company have sufficient storage capacity to store its products [5], which was confirmed by company. Both policy use a service level of 95%, causing an expected annual inventory cost represented by the following equations:

$$\text{Total Cost Policy 1}: 12C_o + \left(\frac{Q_m}{2}\right)C_h + (\sigma_L \cdot Z_a)C_h \quad (11)$$

$$\text{Total Cost Policy 2}: \left(\frac{D}{Q}\right)C_0 + \left(\frac{Q}{2}\right)C_h + (\sigma_L \cdot Z_a)C_h \quad (12)$$

As already stated, the total cost of each policy is also compared for each approach in the calculation of the values of S_L (see section 5). These results are shown in the next section.

RESULTS AND ANALYSIS

First of all, a descriptive summary for the historical data in Table 1 is done to checking the type of demand for each product, these results are shown in Table 3. The skewness and kurtosis values confirm that the demands of the products follow a standard normal distribution with parameters $(\bar{x}, \sigma_s^2)$. Note that the CV value obtained for Styrofoam dishes

Table 3. Parameter $\{\bar{x}, \sigma_s^2, CV, \text{kurtosis and skewness}\}$ per product type.

Parameter	Styrofoam dishes	Rolls of plastic sheeting	Die-cut bags
$\bar{x}$	3077.02	15.29	160
σ_s	582.03	3.66	45.02
σ_s^2	338753.68	13.36	2026.42
CV (%)	18.92	23.90	28.21
kurtosis	0.383	0.059	0.131
skewness	0.646	0.648	0.639

is 18.92%; however, due to the proximity to the reference value of 20% (see section 5 again), this demand is assumed as probabilistic.

Minimum values for the mean squared error (MSE) by using the MA, WMA and ES methods are shown in Table 4. Note that the MA and WMA methods were tested for values of $K + 1$ between 2 and 8, and 2 and 6, respectively (see Tables 5 and 6). The β_i values used in the WMA method are also shown in Table 6. These experiments were compared with each other, selecting the $K + 1$ value that minimizes the average of the MSE for each of the products. In this case, the outputs confirm an optimal value $K + 1$ of 3 and 4 for Styrofoam dishes and Die-cut bags in the WMA method (see Demand vs forecast demand in Figures 2 and 4, respectively). By using the algorithm Solver for Microsoft Excel, it was obtained an optimal value of $q = 0.2$ for the Rolls of plastic sheeting in the ES method (see Demand vs forecast demand in Figure 3).

The values for the forecasted demand, the absolute forecast error and the mean absolute error for the best forecasting methods are displayed in Table 8. Also, the outputs for σs, σ_f and σ_a by using eqs. (7), (8) and (9) are shown in Table 7. These values will allow us to calculate the total cost for each policy proposed in section 6 and represented by eqs. (11) and (12). In cases where it implies to use the deviation for the historical monthly data $\left(\sigma_s \text{ and } \sigma_a \text{ with } \sigma_L = \sigma_s / \sqrt{30} \cdot \sqrt{L}\right)$ or $\sigma_L = \left(\sigma_a / \sqrt{30} \cdot \sqrt{L}\right)$, the average value for the historical monthly demand $\bar{x}$ is used to calculate the Q_m and D values, that is, $Q_m = \bar{x}$ and $D = 12 \cdot \bar{x}$. In

Table 4. Minimum values for the MSE by using the MA, WMA and ES methods.

Type product	MA	WMA	ES
Styrofoam dishes	387614.32	331323.66	342005.87
Rolls of plastic sheeting	15.53	15.71	14.10
Die-cut bags	2123.72	2006.64	2287.41

Table 5. Values for the MSE by using the MA method.

k + 1	K + 1	Rolls of plastic sheeting	Die-cut bags
2	394591.96	18.03	2376.63
3	387614.32	16.37	2196.54
4	397256.14	15.73	2123.72
5	449595.85	15.53	2300.65
6	488416.16	14.83	2549.14
7	497697.60	14.98	2486.26
8	484225.10	15.58	2373.63

Table 6. Values for the MSE by using the WMA method.

k + 1	β_i	Styrofoam dishes	Rolls of plastic sheeting	Die-cut bags
2	60% - 40%	370795.96	17.51	2295.83
	70% - 30%	353873.21	17.36	2266.46
3	50% -30% - 20%	352129.93	16.08	2058.73
	65% - 20% - 15%	331323.66	16.14	2054.01
4	45% - 25% - 20% - 10%	351967.51	15.53	2006.64
5	40% - 25% - 20% - 10% - 5%	372806.49	15.31	2067.36
	45% - 25% - 15% -10% - 5%	363555.52	15.43	2046.67
6	40% - 25% - 15% - 12% - 5% -3%	381016.49	15.32	2116.43
	35% - 25% - 15% -12% - 8% - 5%	396958.00	15.71	2164.88

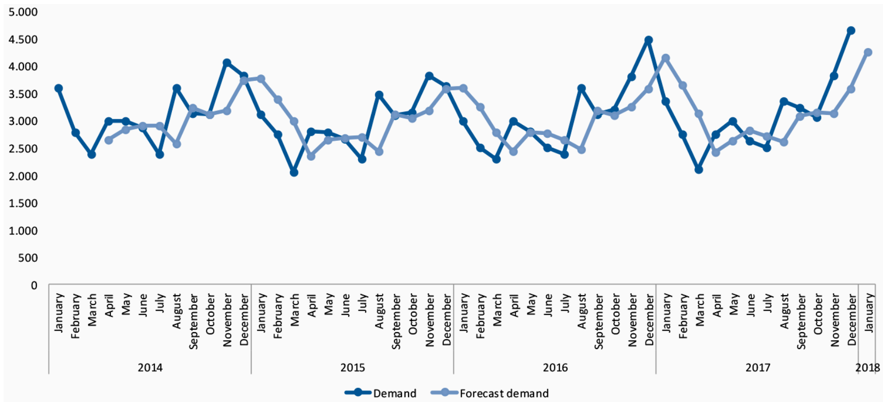

Figure 2. Demand vs forecast demand for Styrofoam dishes and $k + 1 = 3$.

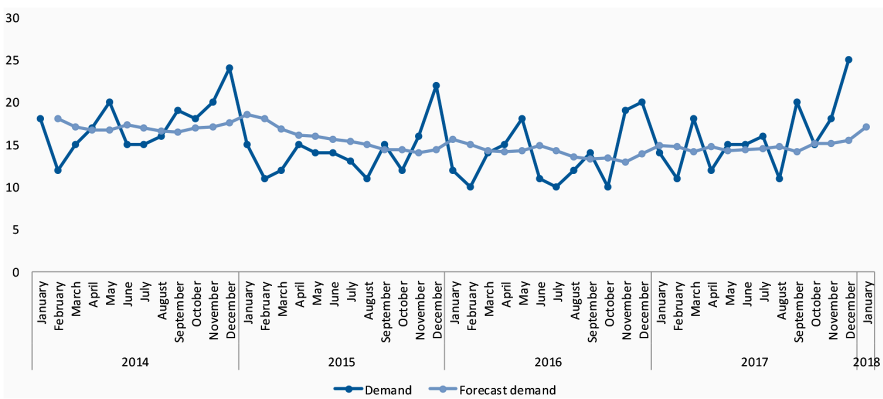

Figure 3. Demand vs forecast demand for rolls of plastic sheeting and $q = 0.2$.

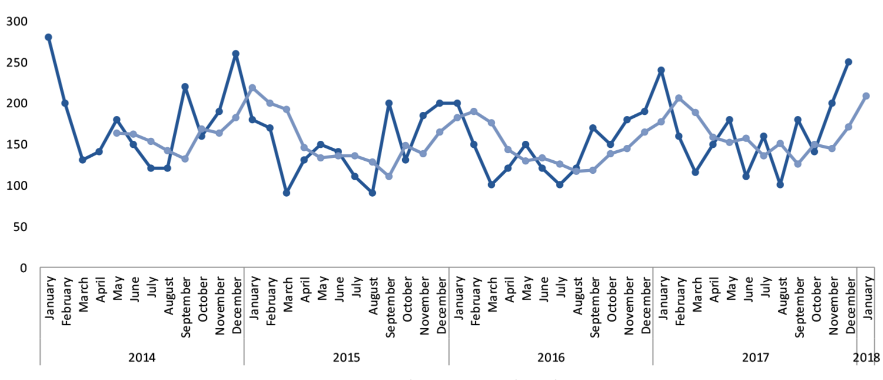

Figure 4. Demand vs forecast demand for die-cut bags and $k + 1 = 4$.

Table 7. Values for σ_s, σ_f and σ_a per product type.

	Styrofoam dishes	Rolls of plastic sheeting	Die-cut bags
σ_s	582.03	3.66	45.02
σ_f	571.91	3.84	46.40
σ_a	561.10	3.59	45.15

Table 8. Forecasted demand, absolute forecast error and mean absolute error for the best forecasting methods.

Year	Month	Styrofoam dishes				Rolls of plastic sheeting				Die-cut bags			
		Demand	Ft (by WMA)	Absolute Forecast Error	Mean Absolute Error	Demand	Ft (by ES)	Absolute Forecast Error	Mean Absolute Error	Demand	Ft (by WMA)	Absolute Forecast Error	Mean Absolute Error
2014	January	3600			522.98	18			2.71	280			120.42
	February	2784			293.02	12	18.00	6.00	3.29	200			40.42
	March	2400			677.02	15	17.04	2.04	0.29	130			29.58
	April	3000	2656.80	343.20	77.02	17	16.71	0.29	1.71	140			19.58
	May	3000	2847.60	152.40	77.02	20	16.76	3.24	4.71	180	163.50	16.50	20.42
	June	2880	2910.00	30.00	197.02	15	17.28	2.28	0.29	150	162.00	12.00	9.58
	July	2400	2922.00	522.00	677.02	15	16.91	1.91	0.29	120	153.50	33.50	39.58
	August	3600	2586.00	1014.00	522.98	16	16.60	0.60	0.71	120	141.50	21.50	39.58
	September	3144	3252.00	108.00	66.98	19	16.51	2.49	3.71	220	132.00	88.00	60.42
	October	3120	3123.60	3.60	42.98	18	16.91	1.09	2.71	160	168.00	8.00	0.42
	November	4080	3196.80	883.20	1002.98	20	17.08	2.92	4.71	190	163.00	27.00	30.42
	December	3840	3747.60	92.40	762.98	24	17.55	6.45	8.71	260	181.50	78.50	100.42
2015	January	3120	3780.00	660.00	42.98	15	18.59	3.59	0.29	180	218.50	38.50	20.42
	February	2760	3408.00	648.00	317.02	11	18.01	7.01	4.29	170	200.00	30.00	10.42
	March	2060	2994.00	934.00	1017.02	12	16.89	4.89	3.29	90	192.50	102.50	69.58
	April	2808	2359.00	449.00	269.02	15	16.10	1.10	0.29	130	145.00	15.00	29.58
	May	2800	2651.20	148.80	277.02	14	15.92	1.92	1.29	150	133.00	17.00	9.58
	June	2664	2690.60	26.60	413.02	14	15.62	1.62	1.29	140	135.00	5.00	19.58
	July	2304	2712.80	408.80	773.02	13	15.36	2.36	2.29	110	135.50	25.50	49.58
	August	3480	2450.40	1029.60	402.98	11	14.98	3.98	4.29	90	127.50	37.50	69.58
	September	3100	3122.40	22.40	22.98	15	14.34	0.66	0.29	200	111.00	89.00	40.42
	October	3150	3056.60	93.40	72.98	12	14.45	2.45	3.29	130	148.50	18.50	29.58
	November	3835	3189.50	645.50	757.98	16	14.05	1.95	0.71	185	137.50	47.50	25.42
	December	3646	3587.75	58.25	568.98	22	14.37	7.63	6.71	200	164.75	35.25	40.42
2016	January	3000	3609.40	609.40	77.02	12	15.59	3.59	3.29	200	182.25	17.75	40.42
	February	2520	3254.45	734.45	557.02	10	15.01	5.01	5.29	150	190.00	40.00	9.58
	March	2300	2784.90	484.90	777.02	14	14.21	0.21	1.29	100	176.00	76.00	59.58
	April	3000	2449.00	551.00	77.02	15	14.18	0.82	0.29	120	142.50	22.50	39.58
	May	2808	2788.00	20.00	269.02	18	14.31	3.69	2.71	150	129.00	21.00	9.58
	June	2515	2770.20	255.20	562.02	11	14.90	3.90	4.29	120	132.50	12.50	39.58
	July	2400	2646.35	246.35	677.02	10	14.27	4.27	5.29	100	125.50	25.50	59.58
	August	3600	2484.20	1115.80	522.98	12	13.59	1.59	3.29	120	117.00	3.00	39.58
	September	3120	3197.25	77.25	42.98	14	13.33	0.67	1.29	170	118.00	52.00	10.42
	October	3214	3108.00	106.00	136.98	10	13.44	3.44	5.29	150	138.50	11.50	9.58
	November	3815	3253.10	561.90	737.98	19	12.89	6.11	3.71	180	144.00	36.00	20.42
	December	4500	3590.55	909.45	1422.98	20	13.87	6.13	4.71	190	164.50	25.50	30.42
2017	January	3360	4170.10	810.10	282.98	14	14.85	0.85	1.29	240	177.50	62.50	80.42
	February	2760	3656.25	896.25	317.02	11	14.72	3.72	4.29	160	206.50	46.50	0.42
	March	2110	3141.00	1031.00	967.02	18	14.12	3.88	2.71	115	188.00	73.00	44.58
	April	2760	2427.50	332.50	317.02	12	14.74	2.74	3.29	150	158.75	8.75	9.58
	May	3000	2630.00	370.00	77.02	15	14.30	0.70	0.29	180	152.25	27.75	20.42
	June	2635	2818.50	183.50	442.02	15	14.41	0.59	0.29	110	157.50	47.50	49.58
	July	2520	2726.75	206.75	557.02	16	14.51	1.49	0.71	160	136.00	24.00	0.42
	August	3360	2615.00	745.00	282.98	11	14.75	3.75	4.29	100	150.50	50.50	59.58
	September	3240	3083.25	156.75	162.98	20	14.15	5.85	4.71	180	125.00	55.00	20.42
	October	3070	3156.00	86.00	7.02	15	15.09	0.09	0.29	140	149.00	9.00	19.58
	November	3840	3147.50	692.50	762.98	18	15.07	2.93	2.71	200	144.00	56.00	40.42
	December	4675	3596.00	1079.00	1597.98	25	15.54	9.46	9.71	250	171.00	79.00	90.42
2018	January		4267.25				17.06				208.50		

Table 9. Outputs for the proposed inventory policies.

	Styrofoam dishes		Rolls of plastic sheeting		Die-cut bags	
	Policy 1	Policy 2	Policy 1	Policy 2	Policy 1	Policy 2
Standard deviation of the historical data						
Inputs						
Demand units (Qm for policy 1 and D for policy 2)	3077.02	36924.24	15.29	183.48	160	1920
Ordering Cost (Co)	1200.00	1200.00	4000.00	4000.00	10000.00	10000.00
Holding cost per unit per year (Ch)	698.40	698.40	36118.80	36118.80	1202.40	1202.40
Lead Time (LT) in days	8	8	8	8	8	8
Standard deviation of the historical monthly data (σs)	582.03	582.03	3.66	3.66	45.02	45.02
Significance level (∝)	0.05	0.05	0.05	0.05	0.05	0.05
Outputs						
Annual cost ($)	1434167.28	594050.78	436414.33	342539.49	262171.69	260856.78
Order quantity (Units)	3077.02	356.21	15.29	6.37	160	178.71
Order cycle time (Days)	30	2.43	30	8.76	30	23.46
Number of orders per year	12	103.66	12	28.78	12	10.74
Reordering level (Units) = μL + ss	1314.91	1314.91	7.19	7.19	80.91	80.91
Security stock (ss)	494.38	494.38	3.11	3.11	38.24	38.24
Demand during the lead time (μL)	820.54	820.54	4.08	4.08	42.67	42.67
Standard deviation during the lead time (σL)	300.56	300.56	1.89	1.89	23.25	23.25
Mean deviation of forecast errors						
Inputs						
Demand units (Qm for policy 1 and D for policy 2)	3029.95	36359.44	15.36	184.30	154.31	1851.68
Ordering Cost (Co)	1200.00	1200.00	4000.00	4000.00	10000.00	10000.00
Holding cost per unit per year (Ch)	698.40	698.40	36118.80	36118.80	1202.40	1202.40
Lead Time (LT) in days	8	8	8	8	8	8
Mean deviation of monthly forecast errors (σf)	571.91	571.91	3.84	3.84	46.40	46.40
Significance level (∝)	0.05	0.05	0.05	0.05	0.05	0.05
Outputs						
Annual cost ($)	1411728.21	586137.37	443174.81	348577.43	260158.37	258408.65
Order quantity (Units)	3029.95	353.48	15.36	6.39	154	175.50
Order cycle time (Days)	30	2.45	30	8.74	30	23.88
Number of orders per year	12	102.86	12	28.85	12	10.55
Reordering level (Units) = μL + ss	1293.77	1293.77	7.36	7.36	80.56	80.56
Security stock (ss)	485.78	485.78	3.26	3.26	39.41	39.41
Demand during the lead time (μL)	807.99	807.99	4.10	4.10	41.15	41.15
Standard deviation during the lead time (σL)	295.33	295.33	1.98	1.98	23.96	23.96
Mean deviation of the historical data						
Inputs						
Demand units (Qm for policy 1 and D for policy 2)	3077.02	36924.24	15.29	183.48	160	1920
Ordering Cost (Co)	1200.00	1200.00	4000.00	4000.00	10000.00	10000.00
Holding cost per unit per year (Ch)	698.40	698.40	36118.80	36118.80	1202.40	1202.40
Lead Time (LT) in days	8	8	8	8	8	8
Mean deviation of the historical monthly data (σa)	561.10	561.10	3.59	3.59	45.15	45.15
Significance level (∝)	0.05	0.05	0.05	0.05	0.05	0.05
Outputs						
Annual cost ($)	1421751.18	581634.68	434266.78	340391.94	262304.47	260989.55
Order quantity (Units)	3077.02	356.21	15.29	6.37	160	178.71

	Styrofoam dishes		Rolls of plastic sheeting		Die-cut bags	
	Policy 1	**Policy 2**	**Policy 1**	**Policy 2**	**Policy 1**	**Policy 2**
Standard deviation of the historical data						
Order cycle time (Days)	30	2.43	30	8.76	30	23.46
Number of orders per year	12	103.66	12	28.78	12	10.74
Reordering level (Units) = μL + ss	1297.14	1297.14	7.13	7.13	81.02	81.02
Security stock (ss)	476.60	476.60	3.05	3.05	38.35	38.35
Demand during the lead time (μL)	820.54	820.54	4.08	4.08	42.67	42.67
Standard deviation during the lead time (σL)	289.75	289.75	1.85	1.85	23.32	23.32

the same way, in cases where it is necessary to use the deviation for forecast errors (σ_f with $\sigma_L = (\sigma_f / \sqrt{30} \cdot \sqrt{L})$), the average value for the predicted monthly demand $\overline{F_t}$ is used to calculate the Q_m and D values, that is, $Q_m = \overline{F_t}$ and $D = 12 \cdot \overline{F_t}$.

The results of the policies are presented in Table 9 for a level of service of 95%, i.e., $a = 5\%$ (value defined by the company). In general, note how policy 2 always offers the lowest values for annual inventory costs, regardless of the type of product. It is observed that this reduction is greater for Styrofoam dishes and Rolls of plastic sheeting when the mean deviation of the monthly historical data (σ_a) is used. Also, taking into account that σ_a is lower in these products. It is also noted that the biggest reduction in costs for Die-cut bags occurs when σ_f is used, although for this product, the differences in costs between the two policies are minimal. This could mean that for the Die-cut bags the policy used would be indifferent.

On the other hand, it is evident the inventory cycles are reduced in policy 2 and, this reduction is considerably greater for Styrofoam dishes. This increases the number of orders per year, which has to be discussed with the company's supplier. It's also noted how the security stock (*ss*) is minimum for cases where the dispersion is smaller.

In conclusion, it is recommended to use an annual inventory policy for all products, which takes into consideration:

- Styrofoam dishes: a Q = 356.21 units, a ss = 476.6 units, a reordering level = 1297.14 units with a total cost of $ 581634.68.
- Rolls of plastic sheeting: a Q = 6.37 units, a ss = 3.05 units, a reordering level = 7.13 units with a total cost of $ 340391.94.
- Die-cut bags: a Q = 175.5 units, a ss = 39.41 units, a reordering level = 80.56 units with a total cost of $ 258408.65.

CONCLUSIONS

Inventories represent an important part of any company due to its implications in the costs of capital invested, storage, maintenance and ordering. These allow to meet internal and external demand, generating high levels of satisfaction when they are well managed. In this research, an optimization approach for independent demand inventory was proposed to establish the best inventory policy in a company specialized in the commercialization of disposable product.

The approach compared three forecasting methods and three different ways of calculating the safety stock by calculating the standard deviation of the historical data, the mean deviation of forecast errors and the mean deviation of the historical data. The total annual cost for two inventory policies were compared using the demand data for three types of products. In the case study, the results confirm that an annual policy, in which the required quantities are defined by a model of economic order quantity and the lowest value of the dispersion is used, would reduce the current inventory costs, guaranteeing a level of service of 95%. The approach evaluated several methods to calculate the predicted demand, the dispersion of the demand (actual or forecasted) and the economic order quantities as factors that directly influence the inventory policy and the total cost. In this sense, we concluded that the mean deviation of forecast errors does not always yield the lowest values for demand dispersion. In conclusion, this study demonstrated that the proposed methodology is practical and easy to use in companies where inventories have probabilistic and independent demand. In future research, it is proposed to study the influence of the proposed approach on items under probabilistic stationary demand.

REFERENCES

[1] A. Pulido-Rojano, A. Villanueva-Polanco, E. Orozco-Acosta y A. Sierra-Altamiranda. "Modelo matemático para la minimización de la escasez de inventarios en cadenas de suministro inestables" (In Spanish). IV Encuentro Iberoamericano de Investigación Operativa y Ciencias Administrativas (IOCA). Santa Cruz de la Sierra, Bolivia. April, 2013.

[2] H. Taha. "Operations Research: An Introduction". Pearson. 10 edition. New York, United States. 2016. ISBN: 978-0134444017.

[3] K. Salas-Navarro, H. Maiguel-Mejía y J. Acevedo-Chedid. "Metodología de Gestión de Inventarios para determinar los niveles de integración y colaboración en una cadena de suministro" (In spanish). Ingeniare. Revista Chilena de Ingeniería. Vol. 25 Nº 2, pp. 326-337. 2017. ISSN: 0718-3291.

[4] A. Pulido-Rojano, J. Daza-Escorcia y F. Narducci-Marin. "Modelo analítico de puntos de reorden con demanda dinámica en el tiempo" (In spanish). XIV Congreso latino iberoamericano de investigación de operaciones (CLAIO). Cartagena, Colombia. September, 2008.

[5] D.R. Anderson, D.J. Sweeney, T.A. Williams, J.D. Camm, J.J. Cochran and M.J. Fry. "Quantitative Methods for Business". Cengage Learning. 13 edition. Boston, United States. 2015. ISBN: 9781285866314.

[6] M. Torres Salazar y P. García Mancera. "Administración de inventarios un desafío para las pymes" (In spanish). Inventio. Vol. 13 Nº 29, pp. 31-38. 2017. ISSN: 2007-1760.

[7] Y. Zhang, G. Hua, S. Wang, J. Zhang and V. Fernandez. "Managing demand uncertainty: Probabilistic selling versus inventory substitution". International Journal of Production Economics. Vol. 196 (C), pp. 56-67. 2018. ISSN: 0925-5273. DOI: 10.1016/j.ijpe.2017.10.001

[8] A. Nodari, J.K. Nurminen and C. Frühwirth. "Inventory theory applied to cost optimization in cloud computing". Proceedings of the 31st Annual ACM Symposium on Applied Computing (SAC16), pp. 470-473. April, 2016. URL: http://dx.doi.org/10.1145/2851613.2851869

[9] M. Reza, K. Behrooz and F.G. Seyyed Mohammad. "Effect of two-echelon trade credit on pricing-inventory policy of non-instantaneous deteriorating products with probabilistic demand and deterioration functions". Annals of Operations Research. Vol. 257, pp. 237-273. 2017. ISSN: 0254-5330.

[10] A.A. Taleizadeh, H. Reza Zarei and B.R. Sarker. "An optimal control of inventory under probablistic replenishment intervals and known price increase". European Journal of Operational Research. Vol. 257, Issue 3, pp. 777-791. 2017. ISSN: 0377-2217. URL: https://doi.org/10.1016/j.ejor.2016.07.041

[11] S. Priyan and R. Uthayakumar. "An integrated production-distribution inventory system involving probabilistic defective and errors in quality inspection under variable setup cost". International Transactions in Operational Research. Vol. 24, Issue 6, pp. 1487-1524. 2017. ISSN: 1475-3995. URL: https://doi.org/10.1111/itor.12202

[12] M. Arias-Vargas. "Impacto en el inventario de seguridad por la utilización de la desviación estándar de los errores de pronóstico" (In spanish). Tecnología en marcha. Vol. 30 Nº 1, pp. 49-54. 2017. DOI: 10.18845/tm.v30i1.3064.

[13] H. Mokhtari. "Economic order quantity for joint complementary and substitutable items". Mathematics and Computers in Simulation. Vol. 154, pp. 34-47. 2018. ISSN: 03784754. DOI: 10.1016/j.matcom.2018.06.004.

[14] L.A. San-José, J. Sicilia, M. González-de-la-Rosa and J. Febles-Acosta. "An economic order quantity model with nonlinear holding cost, partial backlogging and ramp-type demand". Engineering Optimization. Vol. 50, Issue 7, pp. 1164-1177. 2018. ISSN: 0305215X. DOI: 10.1080/0305215X.2017.1414205.

[15] K. Skouri. "An EOQ model with backlog-dependent demand". Operational Research. Vol. 18, Issue 2, pp. 561-574. 2018. ISSN: 11092858. DOI: 10.1007/s12351-016-0279-0.

[16] I. Krommyda, K. Skouri and A.G. Lagodimos. "A unified EOQ model with financial constraints and market tolerance". Applied

Mathematical Modelling. Vol. 65, Issue 1, pp. 89-105. 2019. ISSN: 0307904X. DOI: 10.1016/j.apm.2018.08.002.

[17] F.A. Nava. "Procesamiento de series de tiempo". Ediciones Científicas Universitarias. México. 2013. ISBN13: 9786071613509.

[18] J.C. Garcia-Diaz. "Predicción en el dominio del tiempo. Análisis de series temporales para ingenieros" (In spanish). Universitat Politècnica de València. Valencia, España. 2016. ISBN: 978-84-9048-438-8.

[19] J. Huang. "Improvement of inventory control and forecast according to activity-based classifications: T company as an example". WSEAS Transactions on Business and Economics. Vol. 14, pp. 38-54. 2017. ISSN: 11099526.

[20] R. Hyndman and G. Athanasopoulos. "Forecasting: principles and practice" 2nd edition. OTexts. Melbourne, Australia. 2018.

[21] B. Billah, M.L. King, R.D. Snyder and A.B. Koehler. "Exponential smoothing model selection for forecasting". International Journal of Forecasting. Vol. 22, Issue 2. pp. 239-247. 2006. URL: https://doi.org/10.1016/j.ijforecast.2005.08.002.

[22] A. Pulido-Rojano, P. Sanchez-Sanchez, y E. Melamed-Varela. "Nuevas tendencias en Investigación de Operaciones y Ciencias Administrativas: Un enfoque desde estudios iberoamericanos" (In spanish). Ediciones Universidad Simón Bolívar. Barranquilla, Colombia. 2018. ISBN: 978-958-5430-88-4.

[23] P. Sanchez-Sanchez, J.R. García-González, C.H. Fajardo Toro, A. Pulido-Rojano, y E. Melamed-Varela. "Simulación de sistemas de emergencia en salud" (In spanish). En: A. Pulido-Rojano, P. Sanchez-Sanchez, y E. Melamed-Varela. (eds.). Nuevas tendencias en investigación de operaciones y ciencias administrativas: Un enfoque desde estudios iberoamericanos, pp. 165-210. Ediciones Universidad Simón Bolívar. Barranquilla, Colombia. 2018.

[24] P. Riquelme, G. Gatica y E. Orozco. "Diseño de un Modelo de Operación para Ruteo de Transporte Urbano Basado en Simulación Discreta" (In spanish). Investigación e Innovación en Ingenierías. Vol. 3 Nº 2. 2015. URL: https://doi.org/10.17081/invinno.3.2.2026.

[25] R. Álvarez Martínez, V. Ávila Díaz y J. Castañeda Villacob. "Herramientas para la gestión de la productividad en la empresa: Experiencias exitosas desde el Caribe colombiano" (In spanish). Ediciones Universidad Simón Bolívar. Barranquilla, Colombia. 2017.