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Newton-Raphson method initialization for non-analytical equations solution linked to anticipated annuities

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Abstract. The series of payments made in equal intervals of time is known, in the world of financial mathematics, as an annuity. An anticipated annuity is one whose periodic payment expires at the beginning of the established payment interval. The non-analytical equation that allows us to calculate the interest rate, linked to the anticipated annuity, can be solved using several numerical methods, in particular, the numerical method called Newton-Rhapson. The main problem with this method is its initialization, which requires of one starting point that, usually, is estimated without any scientific background or using random or arbitraries mechanisms. In order to address this problem, in this paper, we establish as main objective to demonstrate that the Newton-Rhapson method can be initialized using only the data, of an anticipated annuity, identified as capital, income and payment intervals without the need to use the initialization strategies, reported in the literature. Through this article, a strategy is presented that allow us to calculate the value of the AA interest rate using the MNR. The value of the error generated for the problematic considered in order to assess the quality of the work performed, is a clear indicator of the good performance of the proposed strategy. This strategy for obtaining the starting point of the aforementioned numerical method is useful in the financial mathematical context, for example, when is necessary the interest rate calculation.

1. Introduction

The interest rate is a price, the price of credit and its level represents currently, the relative scarcity of capital in an economy. The series of payments made in equal intervals of time is known, in the world of financial mathematics, as an annuity. An anticipated annuity (AA) is one whose periodic payment expires at the beginning of the established payment interval [1].

It is a known fact that if the money is invested or financed, during a period of time and at a certain interest rate, it generates a profit or interest. The calculation of the interest rate, which governs the AA, involves the use of a non-analytical equation that requires the application of numerical techniques to obtain the value of the aforementioned rate. Particularly, the numerical method called Newton-Rhapson method (NRM) is a useful tool for the interest rate calculation. In this sense, as a first alternative, people use pre-defined financial tables for the development of problematic situations related to interest rate. The main limitation of this method lies in the fact that it allows the calculation of interest rates only for the parameters that appear tabulated, but if the "data" of the problematic situation to be resolved do not correspond to the values contained in them, this alternative it cannot be applied [2].

Another method is based on an arbitrary initial guess used for a human or material operator who must calculate the interest rate. This method is known as the riddle mechanism. Since the assumption is arbitrary, it lacks justification and financial basis and its main limitation and the alternative is represented by the use of financial calculators. Its main limitation is that it gives wrong answers when n is fractional [3].

In other references such as [4-8], a graphical heuristic method is considered however this method is very time consuming and it has, as additional problem, the incompatibility of the scales produced by the no-linearity of the mathematical model that govern the anticipate annuity.

In this point, it is important say that the NRM requires one initialization value, usually by default value (I_{def}), necessary for the interest rate calculation. In this sense, the main objective, in this article, is proposed to use a model consisting of a novel equation for initial value of interest rates in AA and a classical NRM for calculation of interest rates.

The useful of this kind of equation is that it is not necessary to apply any of the methodologies reported in literature for starting point calculation required by the NRM initialization but that this method can be initialized considering only the known data in the financial mathematical context relate to anticipates annuities.

2. Method

2.1. Understanding the AA mathematical model

A mathematical model that governs the AA is given by Equation (1).

$$P = A + \frac{A[-1+(1+i)^{n-1}]}{i(1+i)^{n-1}}, \tag{1}$$

being: P the current value (Capital), A payment per period (Income), n the time interval of the AA and i the interest rate of the AA. According with the reference [2], this mathematical model represents a non-analytical equation. So, it is impossible to calculate the interest rate using the classical analytic methodologies. For this reason, in this paper, we development a new strategy oriented to interest rate calculation. This strategy is explained at next. If we apply arithmetic sum fractions in the right side of Equation (1), we obtain the Equation (2).

$$P = \frac{A[i(1+i)^{n-1}] + A[-1+(1+i)^{n-1}]}{i(1+i)^{n-1}} \tag{2}$$

Apparently, Equation (2) exhibits a more complex structure that Equation (1) but this complexity is necessary in order to generate a mathematical model that let us to build a real function for calculating the starting point required by NRM initialization. At next, this equation will be modify doing use of factorization techniques for generating the Equation (3).

$$P = \frac{A[i(1+i)^{n-1} - 1 + (1+i)^{n-1}]}{i(1+i)^{n-1}} \tag{3}$$

In Equation (3), the duplicity of the parameter A has disappeared. If we pass this parameter to the left side of the Equation (3) then we can obtain the Equation (4).

$$\frac{P}{A} = \frac{i(1+i)^{n-1}}{i(1+i)^{n-1}} - \frac{1}{i(1+i)^{n-1}} + \frac{(1+i)^{n-1}}{i(1+i)^{n-1}} \tag{4}$$

In the Equation (4) right side it is possible appreciate an implicit mathematical model for the interest rate. Now, we apply simplification techniques in the right side of Equation (4) for obtaining the Equation (5).

$$\frac{P}{A} = 1 - \frac{1}{i(1+i)^{n-1}} + \frac{1}{i} \tag{5}$$

This equation represents a no lineal model for the interest rate linked to anticipate annuity. If now we eliminate the denominators in the right side of Equation (5), we obtain the Equation (6).

$$i * [\frac{P}{A} - 1] = [1 - (1+i)^{-n+1}] \tag{6}$$

Notice the simpler structure of Equation (6) compared with the original Equation (1). Finally, we construct a real function which independent variable is the interest rate generating the Equation (7). This equation is employed for calculating the starting point of NRM.

$$f(i) = i * \left[\frac{P}{A} - 1\right] - [1 - (1+i)^{-n+1}], \tag{7}$$

where: f(i) is the definition of a function whose domain is given by the real numbers comprise between 0 and 1. This equation is vital for generation of mathematical model for calculating the interest rate of AA.

2.2. Obtaining the mathematical models for Newton-Rhapson method initialization (I_{def} value)

Using elements of the optimization theory [9-12], Equation (8) is obtained. Newton-Rhapson is a method based on the first derivative of a function [8]. For this reason, this equation represents the mathematical model for the first derivative initialization using the starting point necessary for calculation of the interest rate of AA.

$$f'(i) = \left(\frac{P}{A} - 1\right) + (-n+1) * [(1+i)^{-n}], \tag{8}$$

being: $f'(i)$ the first derivative of f(i). If equation (8) is equaled to zero and we make some algebraic manipulations Equation (9) can be obtained. This equation represents the mathematical model for the value by default of the interest rate (i_{def}). Notice that the i_{def} mathematical model only depend of the know parameters P, A and n.

$$i_{def} = [\left(\frac{P}{A} - 1\right) * \frac{1}{(n-1)}]^{\frac{1}{(-n)}} - 1 \tag{9}$$

2.3. The mathematical models for Newton-Rhapson method

NRM allows you to determine by approximation an unknown value using an initial approximation for the unknown variable. The NRM is governed by the Equation (10) and it is adequate for the interest rate calculation [8].

$$i_{j+1} = i_j - \frac{f(i_j)}{f'(i_j)}, \tag{10}$$

where: j belongs to the natural numbers set included the zero value as first value. For example, if j is zero both $f(i_j) = f(i_0)$ and $f'(i_0) = f'(i_{0j})$ are real functions assessed in zero, that is, as an initial step the NRM is initialized considering as i_0 the value obtained for i_{def} using Equation (9).

3. Results

By way of example, a problematic situation (PS) is presented below that allows to exemplify the use of the method proposed in this article. PS: There is an obligation of $ 212,491.72 that had been agreed with 18 equal installments of $ 15,000 each per month in advance. It is decided at the last minute to cancel them in cash. What is the monthly interest rate for the year? Use an error of 0.01 as value for the stop criterion. Using as premise that the solution of PS requires of starting point calculation for the NRM initialization, at next it is development the step by step of PS solution.

So, we can calculate this starting point considering only the data of the PS linked to the AA, which given by: The capital P=$ 212,491.72, the income A = $ 15,000 and the time interval n =18 months. Now, using the Equation (9) we obtain that $i_{def} = i_o$= 0.042898. Then i_o= 0.042898 is the starting point required for Newton-Rhapson method for its initialization. Using this starting point and an excel datasheet we can calculate *i* that is the interest rate of the AA. As result we generate the Table 1, which shows the results about the iterations for calculating the unknown monthly interest rate (i), using NRM initialized with the novel Equation (9) proposed in this article.

Table 1. Results about the problematic situation proposed using NRM.

P	A	n	i_dato	i_j	F (i_j)	f' (i_j)	i with NRM
212491.72	15000	18	0.03	0.042898	0.054455	5.184447	0.032394
212491.72	15000	18		0.032394	0.008109	6.757784	0.031194

In this exercise, the NRM is stopped in two iterations because the absolute relative error reaches the value 0.01 established as the reference value for the stop criterion, when it is considered first and second values for i.

In this section it is important point out that the real contribution of this paper is just the calculation of the starting point without to use the classical methodologies, reported in the specialized literature, such as: pre-defined financial tables, the riddle mechanism or graphical methods. Additionally, one of the advantages of Equation (9) for starting point calculation is the huge number of significant numbers that its structure can generate let us consider an important number of decimal values necessaries when is required high precision in some particular applications.

4. Conclusions

Through this article, a novel strategy is proposed for calculating the starting numerical point to initialize a numerical method which is used to calculate the interest rate linked to the anticipate annuities. For this, optimization theory elements and Newton-Rhapson method were used in order to obtain the starting value necessary for NRM initialization.

The main advantage of this strategy is the generation of the mathematical model, for calculation of the starting point, which is given by the Equation (9). This equation is not reported in the specialized literature and, for this reason, it constitutes a novel model for the starting point calculation not only for NRM but for any other numerical method that requires one starting point and that can be used in the financial mathematics context.

In the future, it is planned to extend the proposed strategy for interest rate calculation linked to another types of annuities in order to estimate the robustness of the aforementioned strategy.

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